

Natural Frequencies of Pressurized Hot Fluid Conveying Pipes

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Abstract— In this study, the transverse natural frequencies of a pressurized hot fluid conveying pipe is investigated using complex mode function. Employing the dispersive relations and the non-trivial solution of the coefficient matrix obtained from the boundary equations, the eigenvalues and the linear natural frequencies are obtained numerically. The parametric study is conducted to highlight the effects of variation in operating pressure and pressure drop on the first two modes of the natural frequency of the system. The natural frequency was found to increase nonlinearly with the increase in the operating pressure and pressures drop but decreases with flow velocity.

Keywords— Fluid-conveying pipe, natural frequency, pressure variation, transverse vibrations

1 INTRODUCTION

Pipes or tubes conveying hot pressurized fluids are of immense use in many engineering applications, such as medical equipment (e.g., blood pressure monitoring transducers). Other areas of applications include oil and gas exploitation, heat exchangers, nuclear reactors, chemical plants and various process plants. There are other new and emerging applications in drug delivery, microfluidic and nanofluidic devices (Paidoussis 1998; Qirke 2007; Whitby and Wang 2009; Wang 2010). The effect of pressure variation cannot be overemphasized in that most of these systems are designed for specific operational range of pressures outside which danger can occur. Design and analysis of structures, such as pipes should take into account the operating conditions that can affect the vibration and integrity of such structures. Examples of such operating conditions are the operating temperature, operating pressure, and differential pressure that are experienced during the conveyance of fluid in pipes.

Several review works have been done by Wickert and Mote (1988) and Chen (2005). Many of the reviewed works are based on elastic models (Thurman and Mote, 1969; Pakdemirli, Ulsoy and Ceranoglu, 1994; Pakdemirli and Ulsoy, 1997; Pakdemirli and Ozkaya, 1998; Pakdemirli, 1999; Vestroni, 2000; Hedrih, 2007; Oz and Olunloyo et al., 2007; Koivurova, 2009; Olunloyo, Oshoku and Adelaja, 2010; Ghayesh, 2011; Adelaja, 2013). The nonlinear analysis of a fluid conveying pipe with simply supported ends was first studied by Thurman and Mote (1969) using small perturbation technique. It was discovered that in evaluating the natural frequencies of the system, the importance of nonlinear terms increases with flow velocity so that the range of application of linear theory becomes restricted as the flow velocity increases. On the study of the principal resonance and combination resonances of any two modes for an axially accelerating string, Pakdemirli and Ulsoy (1997) found that there were instabilities when the fluctuating frequency was near twice any natural frequency, but there were no instabilities for frequencies close to zero. Pellicano and Zirilli (1998) applied boundary layer solution for the axially moving beam with small flexural stiffness.

Oz and Pakdemirli (1999) investigated the stability of an axially accelerating elastic tensioned beam moving with harmonically varying velocity. The method of multiple scales was employed in the analysis of the equation of motion, and the influence of small fluctuations in velocity on the stability of the system was investigated. Oz and Boyaci (2000) applied direct perturbation method for the problem of transverse vibration of tensioned fluid conveying pipes with time-dependent velocity. Operational methods were employed in the study of the dynamic behavior of pipeline laid on the seabed (Olunloyo, et al., 2007; 2010). Adelaja (2013), on the study of the temperature modulation of the dynamic responses of flexible fluid conveying pipes, applied the hybrid Fourier-Laplace transforms method. However, the Coriolis acceleration effect was not properly evaluated because of the mixed term. Ghayesh (2011) investigated the effects of axial speed and ply orientation angle on the natural frequencies, complex mode functions, and critical speeds of axially moving laminated composite beams. The parametric study was also done on the effects of system variables on the vibration characteristics of the system.

Hermansen and Thomsen (2018) employed mode shape and damping coefficient methods to develop estimation for the boundary parameter for elastic beam using measured natural frequencies. Both methods can be combined to estimate the effects of boundary tension and boundary damping on the natural frequencies and damping ratios. The methods accurately estimated the natural frequencies, mode shape coefficient close to the measured values. Tan et al. (2018) investigated the equilibrium configurations and natural frequencies of Timoshenko pipe conveying fluid on the supercritical regime. The effects of length and thickness on mid-point deformation of the Euler-Bernoulli and Timoshenko pipes were compared. Timoshenko pipe was said to reach equilibrium bifurcation faster.

In this paper, the solution of the nonlinear transverse vibration equation was obtained using the complex mode function. This method has been found to handle the mixed term (i.e., Coriolis force) conveniently compared with the hybrid Fourier-Laplace transformation technique used by Adelaja (2013). The pressurized, hot fluid is mathematically modeled together with the pipe-foundation system and solved

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with the technique mentioned above to obtain the natural frequencies.

2 THE GOVERNING DIFFERENTIAL EQUATIONS

The specific problem under investigation consists of a hot fluid conveying pipe subjected to pre-stress and fluid pressure as it rests on an elastic Winkler foundation as presented in Fig. 1. The full derivation of the governing differential equation with the boundary conditions and assumptions can be found in the literature (Adelaja, 2011, 2013).



Fig. 1: Schematic diagram of a simply supported pipe

The transverse equation is presented as

$$m\ddot{w} + C_1\dot{w} + 2m_f U\dot{w}' - (T_0 - pA - m_f U^2 - \alpha EA'_t \Theta)w'' + (p'A + pA' + \alpha EA'_t \Theta + \alpha EA_t \Theta')w' + EIw^{IV} + F_1(t) = 0 \quad (1)$$

where $F_1(t)$ is the forcing function term in equation (1) and it is expressed as the foundation parameter in this case. E is Young's modulus of elasticity of the pipe; α is the coefficient of thermal expansion; A and A_t are the cross-sectional area of fluid and pipe respectively and Θ is the temperature of the system. m_f is the mass of fluid per unit length, m is the total mass of the fluid and pipe per unit length, U is the fluid velocity, and I is the moment of inertia. T_0 is the pipe tension; p is the fluid pressure; C_1 is the damping force per unit velocity; u and w are the axial and transverse displacements respectively. $[\square]'$... $[\square]^{IV}$ are the first to the fourth order spatial derivatives. $[\square]$ and $[\square]'$ are the first and second-time derivatives while $[\square]'$ is the mixed term – the Coriolis term. The analysis of the transverse vibration problem is presented in the next section.

3 ANALYSIS OF PROBLEM

In particular, substituting for $F_1(t) = k_b w$ in equation (1) for the case of a pipe on a Winkler foundation gives

$$m\ddot{w} + C_1\dot{w} + 2m_f U\dot{w}' - (T_0 - pA - m_f U^2 - \alpha EA'_t \Theta)w'' + (p'A + pA' + \alpha EA'_t \Theta + \alpha EA_t \Theta')w' + EIw^{IV} + k_b w = 0 \quad (2)$$

where k_b is the soil stiffness. For the case of constant cross-sectional area ($A' = A_t = 0$). Other parameters are defined as follows: $p = p_{ave}$, $\Theta = \Theta_{ave}$, $\Theta' = \Delta\Theta$ and $p' = \Delta p$, where p_{ave} and Θ_{ave} are the average operating pressure and average temperature in the fluid-pipe system respectively, Δp and $\Delta\Theta$ are the total pressure drop (pressure difference) and temperature difference between the inlet and exit sections of the pipe respectively. Substituting these into equation (2) reduces it to

$$m\ddot{w} + C_1\dot{w} + 2m_f U\dot{w}' - (T_0 - pA - m_f U^2 - \alpha EA'_t \Theta)w'' + (\Delta pA + \alpha EA_t \Delta\Theta)w' + EIw^{IV} + k_b w = 0 \quad (3)$$

On introducing the following dimensionless quantities into equation (3)

$$\bar{x} = \frac{x}{L}, \quad \bar{w} = \frac{w}{L}, \quad \delta = \frac{m_f}{m}, \quad \bar{t} = \sqrt{\frac{EI}{m}} \frac{t}{L^2}, \quad \bar{U} = UL\sqrt{\frac{m_f}{EI}}, \quad \bar{A} = \frac{A}{L^2}$$

$$\beta_o = \frac{T_0 L^2}{EI}, \quad \bar{p} = \frac{pL^4}{EI}, \quad \Delta\bar{p} = \frac{\Delta p L^4}{EI}, \quad \bar{C}_1 = \frac{c_1 L^2}{\sqrt{mEI}}, \quad \varepsilon_i = \alpha\Theta, \quad \beta_1 = \frac{EA_t L^2}{EI}, \quad \bar{k}_b = \frac{k_b L^4}{EI}$$

The dimensionless form of the governing equation for the transverse vibration becomes

$$\frac{\partial^4 \bar{w}}{\partial \bar{x}^4} - (\beta_o - \bar{p}\bar{A} + \bar{U}^2 - \varepsilon_i \beta_1) \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + 2\sqrt{\delta}\bar{U} \frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{t}} + (\Delta\bar{p}\bar{A} + \Delta\varepsilon_i \beta_1) \frac{\partial \bar{w}}{\partial \bar{x}} + \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + \bar{C}_1 \frac{\partial \bar{w}}{\partial \bar{t}} + \bar{k}_b \bar{w} = 0 \quad (4)$$

Equation (4) is subjected to the pinned-pinned end boundary conditions, namely

$$\bar{w}(0, \bar{t}) = \bar{w}(1, \bar{t}) = 0$$

$$\bar{w}_{xx}(0, \bar{t}) = \bar{w}_{xx}(1, \bar{t}) = 0 \quad (5)$$

4 SOLUTION METHOD

To calculate the natural frequency, for the homogeneous equation, let the general solution be of the form

$$\bar{w}_n(\bar{x}, \bar{t}) = \sum_{n=1}^{\infty} [e^{i\bar{\omega}_n \bar{t}} \bar{Y}_n(\bar{x}) + e^{-i\bar{\omega}_n \bar{t}} \bar{Y}_n^*(\bar{x})] \quad (6)$$

Where, $\bar{\omega}_n$ is the n^{th} linear natural frequency and $\bar{Y}_n(\bar{x})$ the n^{th} complex mode function, $\bar{Y}_n^*(\bar{x})$ the n^{th} complex conjugate of the term. Substitute equation (6) into equation (4), in terms of the complex mode function, gives

$$\frac{\partial^4 \bar{Y}_n(\bar{x})}{\partial \bar{x}^4} - (\beta_o - \bar{p}\bar{A} + \bar{U}^2 - \varepsilon_i \beta_1) \frac{\partial^2 \bar{Y}_n(\bar{x})}{\partial \bar{x}^2} + 2\sqrt{\delta}\bar{U}i\bar{\omega}_n \frac{\partial \bar{Y}_n(\bar{x})}{\partial \bar{x}} + (\Delta\bar{p}\bar{A} + \Delta\varepsilon_i \beta_1) \frac{\partial \bar{Y}_n(\bar{x})}{\partial \bar{x}} - \bar{\omega}_n^2 \bar{Y}_n(\bar{x}) + \bar{C}_1 i\bar{\omega}_n \bar{Y}_n(\bar{x}) + \bar{k}_b \bar{Y}_n(\bar{x}) = 0 \quad (7)$$

Equation (7) can be re-arranged as

$$\frac{\partial^4 \bar{Y}_n(\bar{x})}{\partial \bar{x}^4} - (\beta_o - \bar{p}\bar{A} + \bar{U}^2 - \varepsilon_i \beta_1) \frac{\partial^2 \bar{Y}_n(\bar{x})}{\partial \bar{x}^2} + (2\sqrt{\delta}\bar{U}i\bar{\omega}_n + \Delta\bar{p}\bar{A} + \Delta\varepsilon_i \beta_1) \frac{\partial \bar{Y}_n(\bar{x})}{\partial \bar{x}} + (-\bar{\omega}_n^2 + \bar{C}_1 i\bar{\omega}_n + \bar{k}_b) \bar{Y}_n(\bar{x}) = 0 \quad (8)$$

Equation (8) is a linear ordinary differential equation with respect to \bar{x} and its solution may be expressed as

$$\bar{Y}_n(\bar{x}) = c_{1n}(e^{i\beta_{1n}\bar{x}} + c_{2n}e^{i\beta_{2n}\bar{x}} + c_{3n}e^{i\beta_{3n}\bar{x}} + c_{4n}e^{i\beta_{4n}\bar{x}}) \quad (9)$$

Where c_{in} ($i = 1, 2, 3, \dots, N$) are different constants.

Substituting equation (9) into equation (8) and differentiating with respect to \bar{x} results in

$$\beta_{jn}^4 + (\beta_o - \bar{p}\bar{A} + \bar{U}^2 - \varepsilon_i \beta_1) \beta_{jn}^2 + (2\sqrt{\delta}\bar{U}i\bar{\omega}_n + \Delta\bar{p}\bar{A} + \Delta\varepsilon_i \beta_1) i\beta_{jn} + (-\bar{\omega}_n^2 + \bar{C}_1 i\bar{\omega}_n + \bar{k}_b) = 0 \quad (10)$$

$\forall i = 1, 2, 3, \dots, m$ and $n = 1, 2, 3, \dots, k$

The boundary conditions for a simply supported pipe with unit length can be expressed as

$$\bar{Y}_n(0)=0, \bar{Y}_n(1)=0, \frac{d^2\bar{Y}_n(1)}{d\bar{x}^2}=0 \frac{d^2\bar{Y}_n(0)}{d\bar{x}^2}=0, \quad (11)$$

Substituting equation (9) into equation (11) gives a set of equations that can be expressed in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ e^{i\beta_{1n}} & e^{i\beta_{2n}} & e^{i\beta_{3n}} & e^{i\beta_{4n}} \\ -\beta_{1n}^2 & -\beta_{2n}^2 & -\beta_{3n}^2 & -\beta_{4n}^2 \\ -\beta_{1n}^2 e^{i\beta_{1n}} & -\beta_{2n}^2 e^{i\beta_{2n}} & -\beta_{3n}^2 e^{i\beta_{3n}} & -\beta_{4n}^2 e^{i\beta_{4n}} \end{bmatrix} \begin{Bmatrix} 1 \\ c_{2n} \\ c_{3n} \\ c_{4n} \end{Bmatrix} c_{1n} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (12)$$

To obtain a nontrivial solution for c_{1n} , c_{2n} , c_{3n} , and c_{4n} the determinant of the coefficient matrix must be zero. This gives rise to the following equation (13) based on the boundary conditions (H. Öz & Pakdemirli, 1999)

$$\begin{aligned} & [e^{i(\beta_{1n}+\beta_{2n})} + e^{i(\beta_{3n}+\beta_{4n})}](\beta_{1n}^2 - \beta_{2n}^2)(\beta_{3n}^2 - \beta_{4n}^2) + \\ & [e^{i(\beta_{1n}+\beta_{3n})} + e^{i(\beta_{2n}+\beta_{4n})}](\beta_{2n}^2 - \beta_{4n}^2)(\beta_{3n}^2 - \beta_{1n}^2) + \\ & [e^{i(\beta_{2n}+\beta_{3n})} + e^{i(\beta_{1n}+\beta_{4n})}](\beta_{1n}^2 - \beta_{4n}^2)(\beta_{2n}^2 - \beta_{3n}^2) = 0 \end{aligned} \quad (13)$$

The set of equations (10) and (13) form five equations with five unknown i.e. β_{1n} , β_{2n} , β_{3n} , β_{4n} and $\bar{\omega}_n$. These can be solved numerically using Newton-Raphson or bisection methods in order to obtain the linear natural frequencies of the system. Substituting these into equation (12) and using elimination process, the coefficients c_{2n} , c_{3n} , and c_{4n} are obtained as

$$c_{2n} = -\frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{3n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{2n}^2)(e^{i\beta_{3n}} - e^{i\beta_{2n}})} \quad (14a)$$

$$c_{3n} = -\frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{2n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{3n}^2)(e^{i\beta_{2n}} - e^{i\beta_{3n}})} \quad (14b)$$

$$c_{4n} = -1 - c_{2n} - c_{3n} \quad (14c)$$

The mode shape function of the system is determined as follows:

$$\begin{aligned} \bar{Y}_n(\bar{x}) = & c_{1n} \left(e^{i\beta_{1n}\bar{x}} - \frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{3n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{2n}^2)(e^{i\beta_{3n}} - e^{i\beta_{2n}})} e^{i\beta_{2n}\bar{x}} - \right. \\ & \frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{2n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{3n}^2)(e^{i\beta_{2n}} - e^{i\beta_{3n}})} e^{i\beta_{3n}\bar{x}} + \left. \left(-1 + \frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{3n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{2n}^2)(e^{i\beta_{3n}} - e^{i\beta_{2n}})} + \right. \right. \\ & \left. \left. \frac{(\beta_{4n}^2 - \beta_{1n}^2)(e^{i\beta_{2n}} - e^{i\beta_{1n}})}{(\beta_{4n}^2 - \beta_{3n}^2)(e^{i\beta_{2n}} - e^{i\beta_{3n}})} \right) e^{i\beta_{4n}\bar{x}} \right) \end{aligned} \quad (15)$$

5 RESULTS AND DISCUSSION

The results and discussion of the analysis of the hot pressurized fluid conveying pipe are presented in this section with a particular interest in the first five modes of the natural frequency and complementary natural frequency. The responses to the investigation of the effects of operating pressure and pressure drop on the natural frequencies for the cases where $\bar{D} = D/L = 3.05 \times 10^{-3}$ and, $\bar{U} = 0 - 12.0$, $\delta = 0.3$, $\beta_0 = 0.3$, $\varepsilon_t = 11 \times 10^{-4}$, $\beta_1 = 0.1$ and $C_1 = 0.1$.

Figs. 2 and 3 depict the variation of the first five modes of the transverse natural frequency and their imaginary components with fluid flow velocity. The parametric study was done on the effects of operating pressure and pressure drop on the first and second modes of the natural frequency. Figs. 2 and 3 show that the first five modes of the natural frequency monotonically decrease

or the imaginary components diverge from the critical velocities. The critical velocity varies between 3.7 and 4.2. The natural frequencies increase with mode and are so ordered such that $\bar{\omega}_1 < \bar{\omega}_2 < \bar{\omega}_3 < \bar{\omega}_4 < \bar{\omega}_5$. These results are in consonance with earlier results obtained in the open literature (Lee & Chung, 2002; H. R. ÖZ & BOYACI, 2000; OzÖz, 2001; Thurman & Mote, 1969).

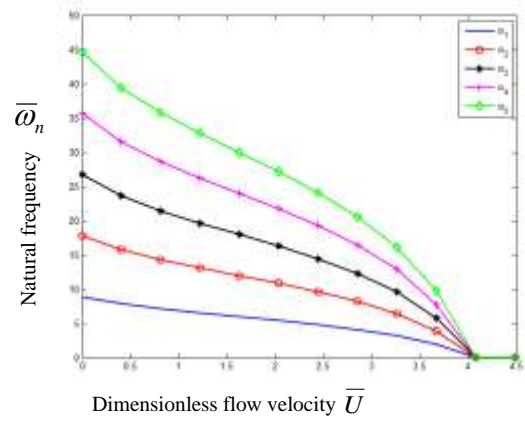


Fig. 2: The variation of the first five real modes of the natural frequency with flow velocity for $\bar{D}_1 = 3.0 \times 10^{-3}$, $\delta = 0.3$, $\bar{p} = 14000$, $\Delta\bar{p} = \bar{p}/20$, $\varepsilon_t = 11 \times 10^{-4}$, $\beta_0 = 0.3$.

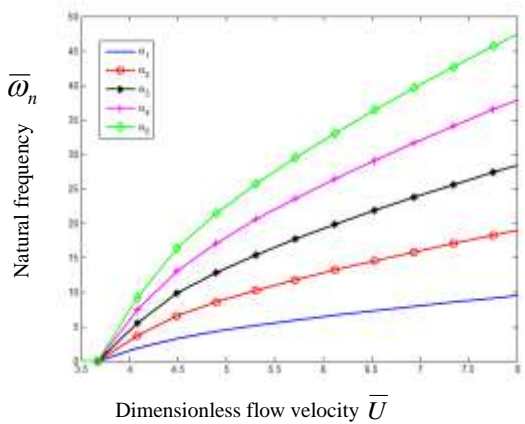


Fig. 3: The variation of the first five imaginary components of the natural frequency with flow velocity for $\bar{D}_1 = 3.0 \times 10^{-3}$, $\delta = 0.3$, $\bar{p} = 14000$, $\Delta\bar{p} = \bar{p}/20$, $\varepsilon_t = 11 \times 10^{-4}$, $\beta_0 = 0.3$.

In Figs. 4 and 5 are shown the variation of the natural frequencies – first and second modes – with flow velocity for different operating pressures (Fig. 4) and pressures drop (Fig. 5). It can be deduced that there is a reduction in the natural frequency as the flow velocity increases. Also, the operating pressure and pressure drop influence the critical velocity such that larger operating and differential pressures correspond to larger critical flow velocities. Furthermore, increase in operating pressure and pressure drop increase the natural frequencies.

Fig. 4 reveals the result of the variation of the first (Fig. 4a) and second (Fig. 4b) modes of the natural frequency with flow velocity for different operating pressure. The two modes of natural frequency increase with an increase in the operating pressure but decrease at varying rates with velocity. The obtained critical velocity ranges between 9.0 and 12.0. Natural frequencies first

increase at early flow velocity between 0.0 and 2.5 for $\bar{p} = 140$ before decreasing to the critical velocities. For higher pumping or operating pressure, there is a nonlinear decrease in the natural frequency with flow velocity.

In Fig. 5, the pressure drop investigated ranged between 0 and 10% of the operating pressure since a large pressure drop is not desirable because of the resulting large pumping power requirement and higher cost. Natural frequencies increase with an increase in pressure drop but first increase at early flow velocity between 0.0 and 0.75 before decreasing to the critical velocities.

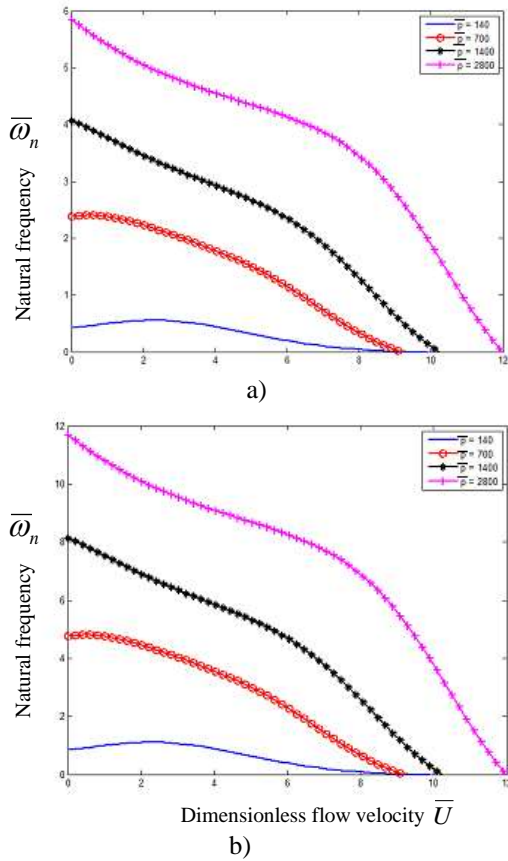


Fig. 4: The variation of the a) first mode and b) second mode of the natural frequency with flow velocity for different dimensionless pressure for small pipe for the case

$$\bar{D}_i = 3.0 \times 10^{-3}, \delta = 0.3, \Delta \bar{p} = \bar{p}/20, \varepsilon_t = 11 \times 10^{-4}, \beta_0 = 0.3.$$

6 CONCLUSION

The transverse natural frequency of the pressurized hot fluid conveying pipe is investigated using complex mode function. Studies were conducted on the effect of the first five modes of natural frequency and the complementary components as the fluid flow velocity increases. Parametric studies were conducted on the effects of operating pressure and pressure drop on the natural frequency. The results show that natural frequency increases with mode as velocity increases. Increase in operating pressure and pressure drop result in a nonlinear increase in the natural frequency. Also, increase in the operating pressure, and pressure drop correspond to an increase in the critical flow velocity. For this study, the critical flow velocity ranged between 3.7 and 12.0.

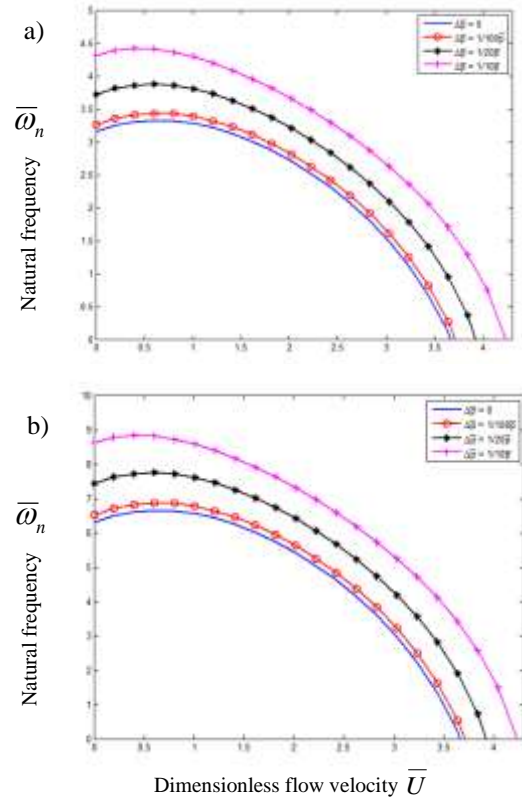


Fig. 5: The variation of the a) first mode and b) second mode of the natural frequency with flow velocity for different dimensionless differential pressure for small pipe for the case

$$\bar{D}_i = 3.0 \times 10^{-3}, \delta = 0.3, \bar{p} = 14000, \varepsilon_t = 11 \times 10^{-4}, \beta_0 = 0.3$$

NOMENCLATURE

A	fluid cross sectional area (m^2)
A_i	cross sectional area of pipe (m^2)
\bar{A}	dimensionless cross sectional area (-)
C_1	damping force/ velocity (Nms^{-1})
\bar{C}_1	dimensionless damping force/ velocity (-)
D	pipe diameter (m)
E	Young's modulus of elasticity (GNm^{-2})
$F_1(t)$	forcing function (N)
g	acceleration due to gravity (ms^{-2})
I	moment of inertia (m^4)
k_b	soil stiffness (Nm^{-1})
\bar{k}_b	dimensionless soil stiffness (-)
L	length of pipe (m)
m	total mass of pipe and fluid/unit length (kgm^{-1})
m_f	mass of fluid/unit length (kgm^{-1})
pA	pressurization effect (N)
\bar{p}	dimensionless pressure (-)
T_o	tension in pipe (N)
t	time (s)
\bar{t}	dimensionless time (-)
u	longitudinal displacement (m)
U	velocity of transported fluid (ms^{-1})
\bar{U}	dimensionless velocity (-)
w	transverse displacement (m)
\bar{w}	dimensionless transverse displ. (-)
x	axial displacement coordinate

- \bar{x} dimensionless axial position (-)
 \bar{Y}_n n^{th} complex mode function
 $[\square]'$ differential with respect to x
 $[\square]$ differential with respect to t

Greek letters

- α coefficient of thermal expansion (K^{-1})
 β_0 dimensionless pipe tension
 β_1 dimensionless flexural rigidity
 β_{ni} Eigenvalues
 δ mass ratio
 $\Delta\theta$ system temperature difference (K)
 Δp differential pressure (Nm^{-2})
 ε_t dimensionless expansion parameter
 $\bar{\omega}_n$ n^{th} linear natural frequency
 Θ system temperature (K)

Subscript

- ave average value

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